

Notes: Solving Trig. Equations

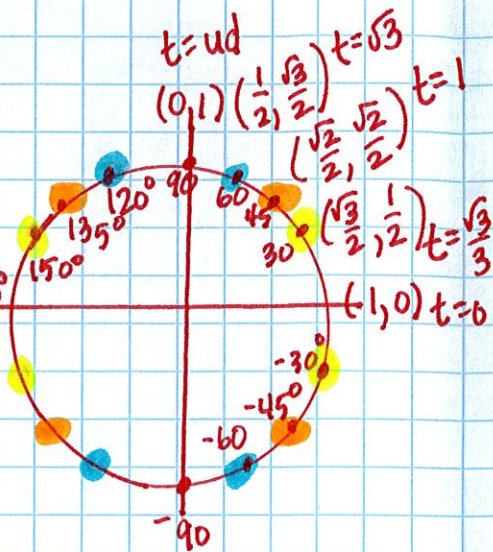
Review: Inverses

A. $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ OR $\frac{\pi}{6}$
 (Q1) Q4

B. $\tan^{-1}(-\sqrt{3}) = -60^\circ$ OR $-\frac{\pi}{3}$
 Q1 (Q4)

C. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$ OR $\frac{5\pi}{6}$
 Q1 (Q2)

D. $\sec^{-1}(2) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ OR $\frac{\pi}{3}$
 (Q1) Q2



Example One Verify that the x-values are solutions of the equation

$$2\sin^2 x + 3\sin x + 1 = 0$$

A. $x = \frac{\pi}{6}$

$$2\left(\sin\frac{\pi}{6}\right)^2 + 3\left(\sin\frac{\pi}{6}\right) + 1 = 0$$

$$2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 = 0$$

$$2\left(\frac{1}{4}\right) + \frac{3}{2} + 1 = 0$$

$$\frac{1}{2} + \frac{3}{2} + 1 = 0$$

$$3 \neq 0$$

False statement

So not a solution

B. $x = 60^\circ$

$$2(\sin 60^\circ)^2 + 3(\sin 60^\circ) + 1 = 0$$

$$2\left(\frac{\sqrt{3}}{2}\right)^2 + 3\left(\frac{\sqrt{3}}{2}\right) + 1 = 0$$

$$\frac{6}{4} + \frac{3\sqrt{3}}{2} + 1 = 0$$

$$\frac{3}{2} + \frac{3\sqrt{3}}{2} + \frac{2}{2} = 0$$

$\frac{5+3\sqrt{3}}{2} \neq 0$ No $x=60^\circ$ is not a solution

Example 2 Solve each equation

A. $2\sin x - 1 = 0$

$$2\sin x = 1$$

$$\sin^{-1} \sin x = \sin^{-1} \frac{1}{2}$$

$$x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = 30^\circ$$

1. To Solve Equation you isolate the variable using inverse operations

2. Find the trig. inverse

3. Could verify by plugging into original.

$$2\sin x - 1 = 0$$

$$2\sin(30^\circ) - 1 = 0$$

$$2\left(\frac{1}{2}\right) - 1 = 0$$

$$1 - 1 = 0$$

$$0 = 0 \checkmark$$

$$\boxed{b.} \quad 3 \tan^2 x - 1 = 0$$

$$\frac{3 \tan^2 x}{3} = \frac{1}{3}$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{\sqrt{1}}{\sqrt{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\cancel{\tan^{-1}} \tan x = \cancel{\tan^{-1}} \pm \frac{\sqrt{3}}{3}$$

$$x = \tan^{-1} \left(\pm \frac{\sqrt{3}}{3} \right)$$

$$x = 30^\circ \text{ and } -30^\circ$$

$$\boxed{c.} \quad \cot x \cos^2 x = 2 \cot x$$

$$\cot x \cos^2 x - 2 \cot x = 0$$

$$\cot x (\cos^2 x - 2) = 0$$

$$\cancel{\cot^{-1}} \cot x = 0$$

$$x = \cot^{-1}(0)$$

$$x = \tan^{-1}(\infty)$$

$$x = 90^\circ$$

$$\cos^2 x - 2 = 0$$

$$\sqrt{\cos^2 x} = \sqrt{2}$$

$$\cancel{\cos^{-1}} \cos x = \cancel{\cos^{-1}} \pm \sqrt{2}$$

$$x = \cos^{-1}(\pm \sqrt{2})$$

$\sqrt{2}$ = bigger than 1
its garbage and
no solution!

$$\boxed{d.} \quad 2 \sin^2 x - \sin x - 1 = 0$$

can rewrite to help you factor

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

$$2 \sin x + 1 = 0 \quad \sin x - 1 = 0$$

$$\frac{2 \sin x}{2} = \frac{-1}{2} \quad \sin^{-1} \sin x = \sin^{-1} 1$$

$$\cancel{\sin^{-1}} \sin x = \cancel{\sin^{-1}} -\frac{1}{2}$$

$$x = \sin^{-1} \left(-\frac{1}{2} \right)$$

$$x = -30^\circ$$

$$x = \sin^{-1}(1)$$

$$x = 90^\circ$$

$$\boxed{e.} \quad 2(\sin^2 x) + 3 \cos x - 3 = 0$$

$$2(1 - \cos^2 x) + 3 \cos x - 3 = 0$$

$$2 - 2 \cos^2 x + 3 \cos x - 3 = 0$$

$$-2 \cos^2 x + 3 \cos x - 1 = 0$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$2x^2 - 3x + 1 = 0$$

$$(2x - 1)(x - 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\frac{2 \cos x}{2} = \frac{1}{2}$$

$$\cancel{\cos^{-1}} \cos x = \cancel{\cos^{-1}} \frac{1}{2}$$

$$x = \cos^{-1} \left(\frac{1}{2} \right)$$

$$x = 60^\circ$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$\cancel{\cos^{-1}} \cos x = \cancel{\cos^{-1}} 1$$

$$x = \cos^{-1}(1)$$

$$x = 0^\circ$$